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Composing from a Geometric Model: *Five-Leaf Rose*

When one considers the vast range of possible applications of mathematics to music, composing from a geometric model would appear to be one of the simpler things to do. Certainly the geometry I have used in composing *Five-Leaf Rose* is conceptually simpler than most applications of set theory or stochastic processes. The purpose of using any such method, of course, is to aid structural integration and unity. What gives this geometric approach useful musical properties is the depth to which it can be applied in a computer-generated composition. The unity is not achieved merely by the idea of a geometric figure, but by having a coordinated pattern of control on many different aspects of the composition. Nearly everything in *Five-Leaf Rose*, from the formal structure to the acoustic details, is tied to this single model. The one exception, melody, is freely composed against the background of these highly organized elements. The geometric figure as such may not be discernible to the audience, but the musical relationships derived from the figure are.

This method of composing might have been conceivable without the use of a digital computer, but would not likely have been carried very far without it. The computer removes all of the practical obstacles to this kind of composition and makes the "conceivable" also possible. The score to *Five-Leaf Rose* was prepared by a composition program that computed the relationship of all of the acoustic details to the large-scale design, a task that would have been extremely unwieldy to do by hand. And, of course, it is only possible to have this kind of control on acoustic details with computer-generated sound.

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Form and Design

The geometric figure of a five-leaf rose has five loops that intersect at a common point, as shown in Fig. 1a. It can easily be drawn with one continuous line looping out and back in, out and back in, until there are five loops and the line returns to the first leaf. There are a number of properties of the five-leaf rose that underlie its use in the design of this composition. One property of the figure is that it is a continuous curve. This property guarantees that the composition will go through slow, continuous changes (rather than have sudden, dramatic contrasts). Another property is that the figure has an underlying pattern; this ensures that the changes in the composition will repeat a pattern. Still another property is that, even though the curve itself is continuous, the angle it traces in relation to the origin has discontinuities that can serve as points of articulation in the composition. As shown in Fig. 1b, as one traces the first leaf beginning with the upper portion, the angle θ moves from 72° to 0°. When one enters the second leaf, the angle jumps to 144° and moves back toward 72° as one traces through the second leaf. This process repeats every 72° until the entire figure is traced. In addition to the discontinuities in the angle, if one imagines that the curve is traced with a pen moving at a constant velocity, the rate of angular change varies and thus provides a large-scale pattern for rates of change that is linked to the shape of each leaf. The rate of angular change is high when one is tracing the most remote part of a leaf, because the curve is cutting directly across the angle. The rate of angular change is low when one is tracing the part of a leaf near the origin, because the curve slopes away gently.

The geometric properties are given musical meaning by assignment of musical parameters to the different dimensions of the graph. In the case of

Fig. 1a. The five-leaf rose.

Fig. 1b. One of the leaves of the rose.

Fig. 2a. A graph showing the density of notes on a leaf.

Fig. 2b. A graph of an imaginary boat (carrying the audience) tracing the curve of a leaf.

Fig. 1a

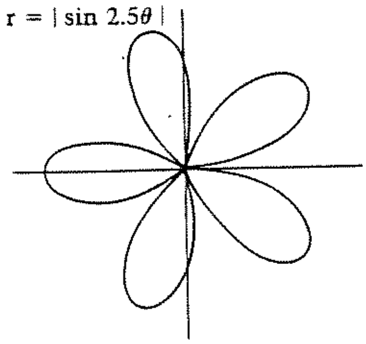


Fig. 1b

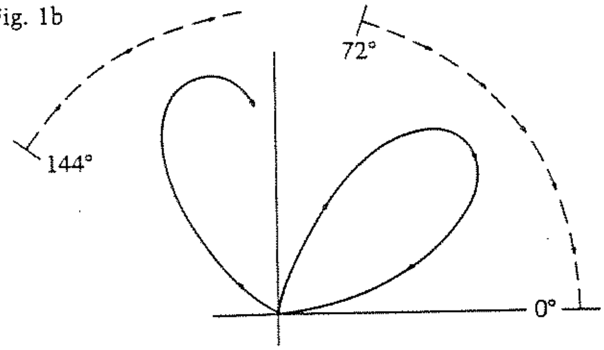


Fig. 2a

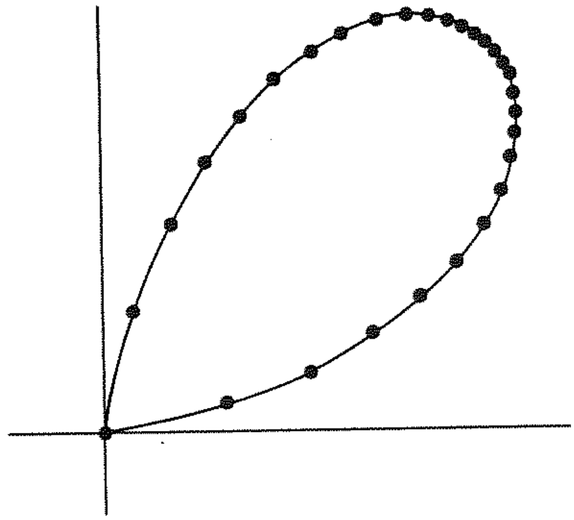
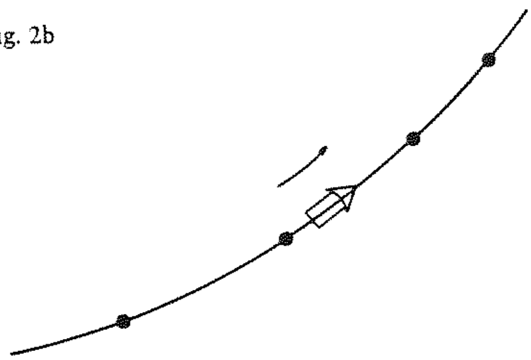


Fig. 2b

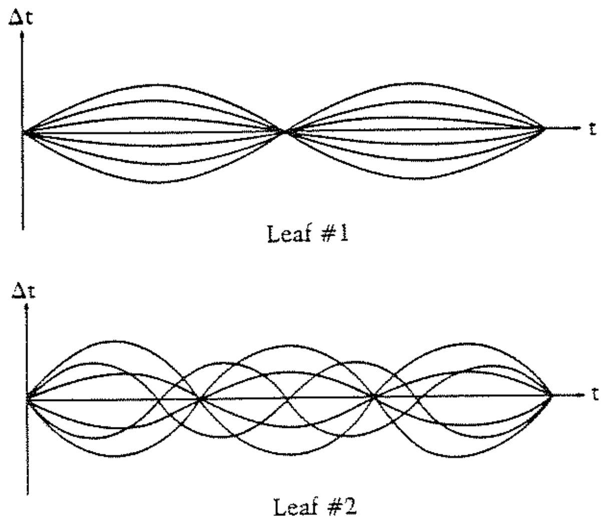


this figure, parameters are assigned according to polar coordinates. The angle θ controls all pitch and timbral aspects of the composition. For example, θ represents a timbral spectrum; all of the possible timbres in the composition are arranged in a circle. If one moved directly around the plane in a circle, the timbres would slowly and continuously modulate back and forth between stringlike and windlike, bright and dark, and so forth (exactly how this is done will be explained in connection with the orchestra program).

The two-dimensional plane itself is also taken as a literal map of a physical plane on which the composition takes place. Each individual note has a unique position on the curve and thus a unique timbre. The notes are positioned, on the average, every 2°, and therefore the density of the notes is greatest in the most remote part of the leaves (Fig. 2a). The form of the composition unfolds not as the sound sources move, but as the audience location moves. It is as if the audience were on a boat tracing the curve of the five-leaf rose, and as the boat moves the notes pass by (Fig. 2b). The speed of the boat changes in such a way that it is fastest as it passes through the origin and it is slowest at the tip of each leaf, somewhat in the manner of a planetary orbit.

One of the consequences of the pitch-timbre scheme is that the composition is based on a single stream of pitches that unfolds as the audience moves around the figure. The actual sounding notes are made to run slightly ahead of and behind the audience's location and thus produce different simultaneous pitches. Six voices sound simultaneously throughout. Whenever the audience's boat passes through the origin, all six sounding voices are also at the origin. But as the audience's

Fig. 3. A graph of time delay versus time for leaves one and two.



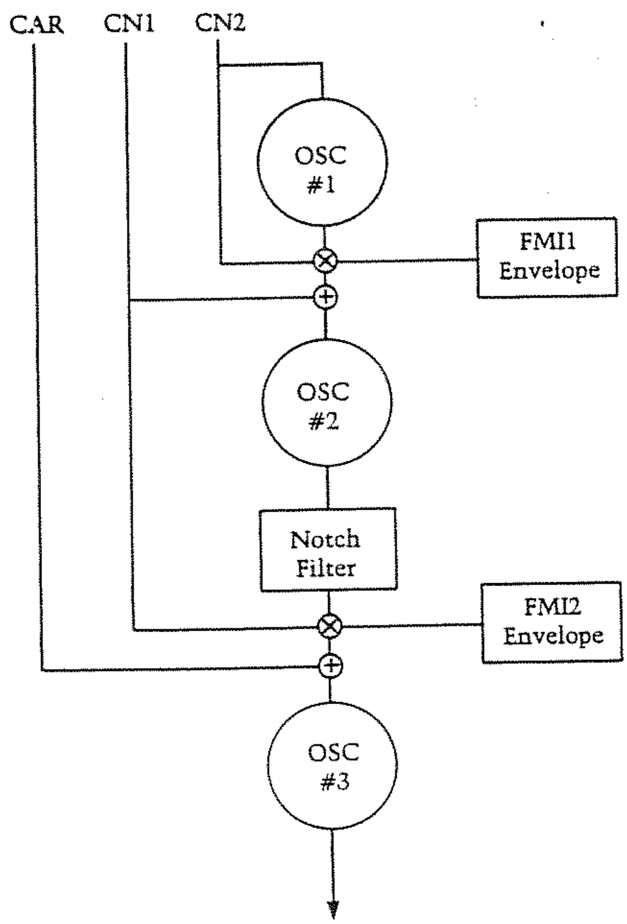
boat moves into a leaf, some voices run ahead and some voices fall behind. The time delay that separates the six voices constantly lengthens and shortens and thus creates a phase canon in both pitch and timbre. Figure 3 is a graph of time delay versus time for leaves one and two. An important consequence of combining the phase canons with the locational scheme is that time delay is translated into physical distance. A given note occurs only at a particular location. As a voice runs ahead of the audience in time, it also runs ahead in location; similarly, as a voice falls behind in time, it is also behind in location. This means that from the audience's perspective, new timbres and pitches come in gradually in front while old timbres and pitches fade away behind—ahead being the future, behind the past.

How do all of these formal elements contribute to the whole? As the piece begins, the audience is at the origin and moves out into the first leaf. As it does so, the six voices slip gradually out of phase with one another. These first notes move past the audience quickly and only gradually do notes begin to move past more slowly. As the audience moves toward the tip of the leaf, the rate of pitch and timbral change accelerates until it reaches its peak at the tip. As the audience moves back toward the origin, there are fewer notes and the voices return to synchronization. As the audience passes through the origin and into the second leaf, there is a sudden discontinuity in pitch and timbre. The second leaf begins to unfold like the first (the rate of sound movement, pitch-timbre change, and spreading of the voices is the same). New pitches and timbres are being introduced, and the phase canon has a different set of time delays. As the audience concludes its movement around the second leaf, the timbral properties become similar to those heard at the beginning of the first leaf. As the audience moves again through the origin, there is a discontinuity in pitch and timbre, and the same relationships among the different elements begin to repeat their patterns. The patterns repeat for each of the five leaves. The form articulated by this scheme is in five parts. Probably the most recognizable repetition within the patterns is the transition from leaf to leaf. (In this respect, the form resembles the construction of the upper voices in many fifteenth-century isorhythmic motets, in which the clearest rhythmic repetition occurs just before each statement of the talea.)

Orchestra, Timbre and Pitch

Almost everything else important to this piece, including the pitch content, derives from the particular design of the single instrument that generates the sounds. The technique used for the timbral generation is a form of frequency modulation (FM). In this case, three oscillators are connected in series, each frequency modulating the next. Although many composers have probably used this scheme for producing complex timbre, it has not been previously mentioned in the literature of digital synthesis as another extension of the FM techniques first discussed by John Chowning (1973). The reason for the omission may be that the modulation of the first two oscillators will often produce a sideband at 0 Hz. When this 0-Hz component is used in modulating the third oscillator, it causes an erratic shift in the resulting carrier frequency. In order to overcome the carrier shift, in front of the third oscillator I inserted a notch filter that removes the 0-Hz component and leaves the rest of the control signal untouched (Fig. 4).

Fig. 4. Simplified diagram of the computer instrument used in Five-Leaf Rose.



This particular FM scheme has many attractive features and is able to produce timbres with realistic qualities. It is particularly good with string timbres, but every type of timbre seems to benefit by having some amount of modulation on the final control signal. This scheme is, of course, capable of producing a wide range of timbres, but within this composition I limited myself to harmonic timbres.

One of the particular advantages of this FM scheme is that it provides yet another dimension to *c*:*m* ratios; they become *c*:*m1*:*m2* ratios. Many more subtle shades of timbre are possible with the three-oscillator scheme than with two; consider that for every *c*:*m* ratio there are numerous *c*:*m1*:*m2* ratios that can either reinforce or fill in the *c*:*m* ratio's spectrum. (One can imagine that an organization of these ratios in terms similar to that done by Barry Truax (1977) for *c*:*m* ratios must be substantially more complex because of the way in which the three oscillators interact.) There are some additional advantages to this scheme when one considers its dynamic properties. With two modulation indexes to manipulate, many subtle differences in types of attacks and slow spectral unfolding are possible. Even better, manipulation of the modulation index for the two control signals provides a simple way to produce a kind of spectral fluctuation resembling that of acoustic instruments. Of course, spectral fluctuation and synchronicity constitute the dimension of timbre described by John Grey (1975) that most closely correlates to the traditional instrument families. It is an extremely important factor in determining timbre and one that is not easy to simulate.

The actual instrument that is used to produce the timbres in *Five-Leaf Rose* is naturally more complex than that shown in Fig. 4. In addition to the parameters involved directly with the double-modulation scheme, there are parameters to control random frequency and amplitude fluctuation, vibrato, detuning of harmonics, low-pass filtering, and so on—a total of 15 parameters that determine timbre. As discussed before, all of these parameters are linked to the angle θ in the geometric model. Each parameter is controlled by a function made up of the addition of two sinusoids like that shown in Fig. 5a. As the angle θ moves from 0° to 360°, the values assigned to the particular acoustic parameter vary in a regular pattern. But, since the angle θ itself unfolds backward through each leaf (72–0°, 144–72°, etc.), the values of the acoustic parameter are also reversed within each leaf, as shown in Fig. 5b. This is the specific way in which the timbral discontinuity at the beginning of each new leaf is achieved.

The functions for all of the acoustic parameters are similar in form. Examples of four functions are shown in Fig. 6. No function is in synchronization with any other, so they all present a slightly different unfolding of the timbral parameters they control. The total effect of all these parameters changing in ensemble is that the timbre continu-

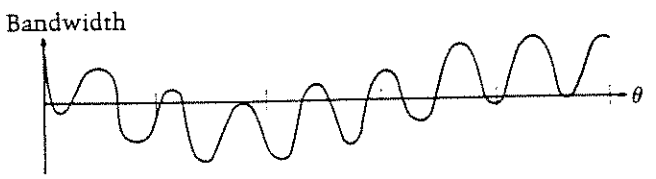


Fig. 5a. The double sinusoid function used to control instrument parameters.

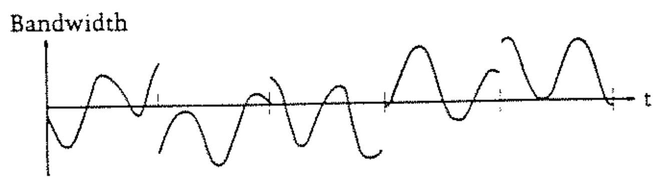


Fig. 5b. Acoustic discontinuity is achieved by reversing the control function at the beginning of each leaf.

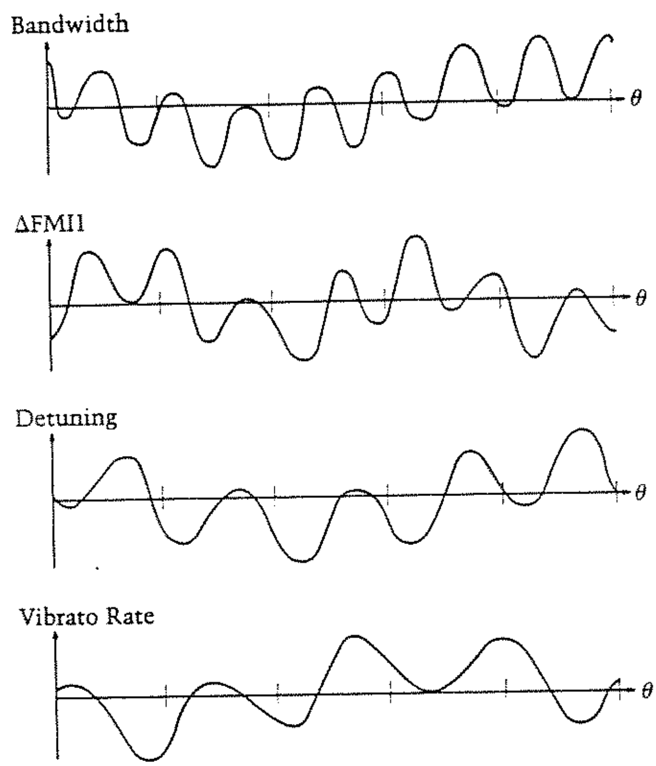


Fig. 6. Examples of four control functions.

ously modulates as θ changes. The combination of functions used in *Five-Leaf Rose* specifies a particular sequence of timbres rather than all possible timbres. The sequence was arrived at through experimentation.

One of the important determinants of timbre is still the $c:m1:m2$ ratio. Because the range of possible ratios is nearly limitless, I adopted a scheme that reduces the choice of ratios to a limited number. Within this scheme the frequency of the middle oscillator is fixed, and a sequence of ratios is created in which $m1$ runs from 1 to 12. The $c:m1$ part of such a sequence would be 1:1, 1:2, 1:3, . . . 1:12 or 1:1, 1:2, 2:3, . . . 11:12, etc. The fundamentals created by any such sequence form an inversion of the harmonic series (Fig. 7); Fig. 8 shows the collections of notes formed for leaves one and five. The ratios automatically map into unique pitches (Table 1). The intervals produced in this way are pure untempered intervals. The menu of available pitches was then expanded by taking each of the original five pitch-classes as the basis for another series (not shown). Each leaf of the piece is based on a scale formed by combining two series; one series is always held in common between adjacent leaves (Table 2).

In spite of the fact that the scales change with each section, the melodic contour is controlled by a smoothly varying function like those controlling the acoustic parameters. After all, because of the direct relationship of pitch to the $c:m1:m2$ ratios, pitch and timbre are inseparable. The function for the melodic contour is shown in Fig. 9a. This function is interpreted more freely than any of the other functions. First, the function is taken as a general determinant of range and not of specific pitches. Second, there are numerous reversals of segments of the function within the individual leaves. These reversals are small reflections of the large-scale reversals that occur between leaves and present a melodic contour with many clearly articulated shapes. Melodic contour versus time is graphed in Fig. 9b.

Melody

For most listeners, the most immediate aspect of the composition is melody. The freedom of melody contrasts with the organization of all other parameters. Whereas all other parameters are linked to the large-scale design, melody creates a spontaneous, moment-to-moment suspense that propels the forward movement. Melody's freedom is tem-

Fig. 7. Fundamentals created by the double modulator scheme.

Fig. 8. Collections of notes generated for leaves one and five.

Fig. 9a. The function for melodic contour.

Fig. 9b. A graph of melodic contour versus time.

Fig. 7

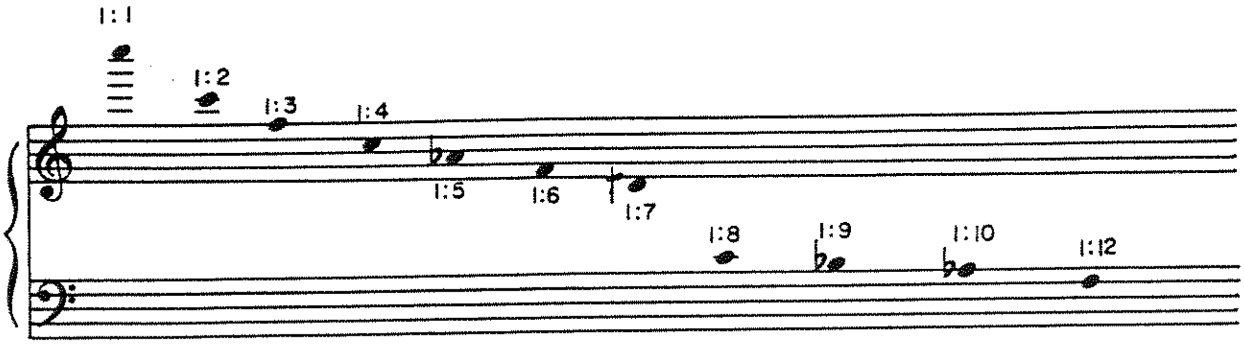


Fig. 8

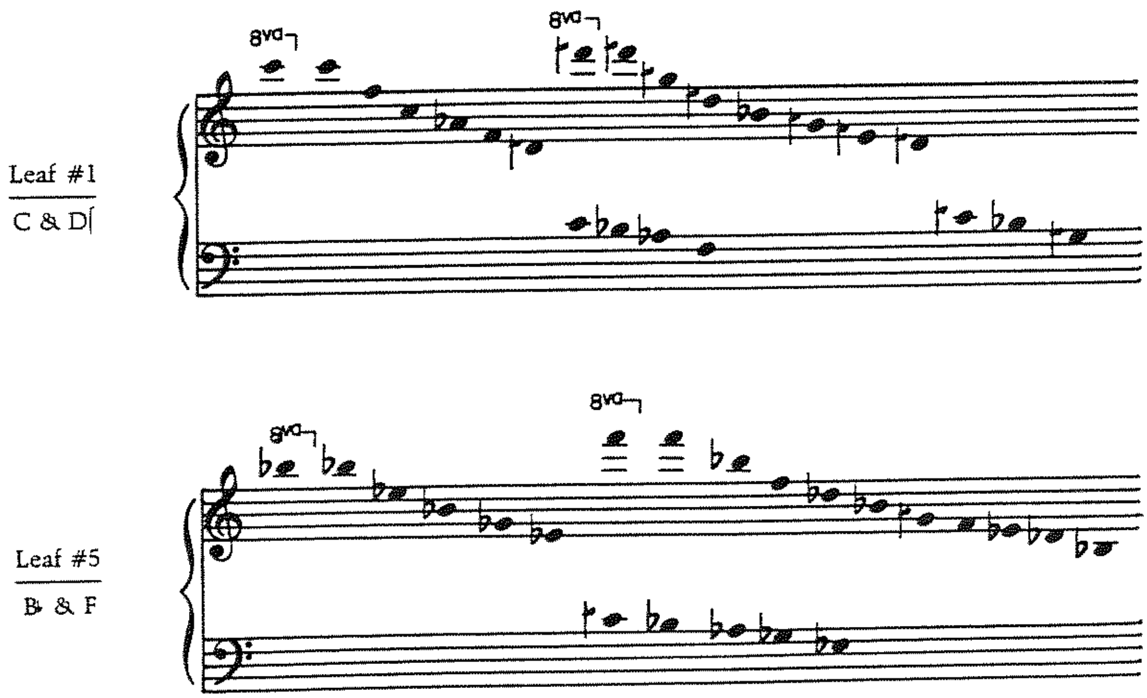



Fig. 9a

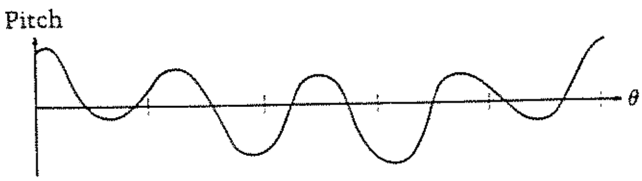


Fig. 9b

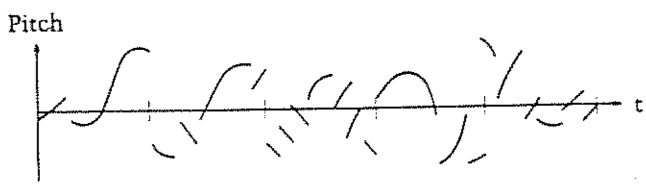
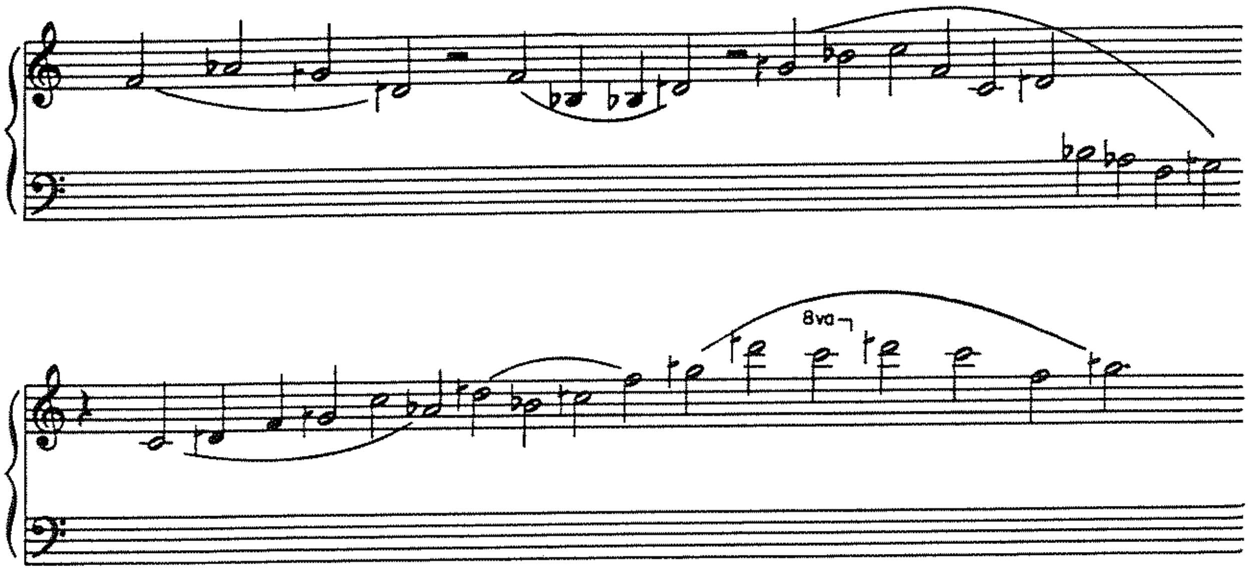


Fig. 10. The melodic lines for leaves one and five.

Leaf #1

Leaf #5

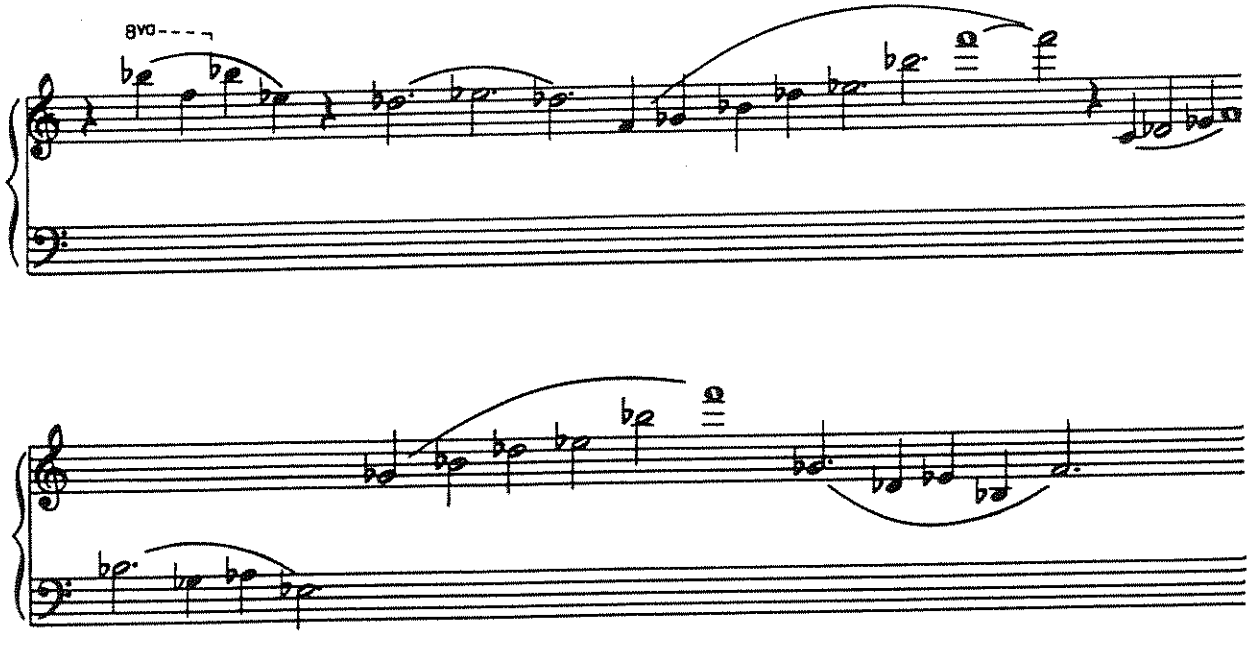



Table 1.

pitch-class	*semitones, cents*
c	0, 00
f	4, 98
a♭	8, 14
d[	2, 31
b♭	9, 96

Table 2.

Leaf #1	C	&	D[
Leaf #2	C	&	A♭
Leaf #3	D[	&	A♭
Leaf #4	D[	&	F
Leaf #5	B♭	&	F

pered by the need to make the phase canons clear, to create harmonic relationships, and to exploit the immediate timbral qualities, but it is largely composed in an intuitive fashion. Sometimes it supports the patterns of the other parameters; sometimes it is in counterpoint to them. Figure 10 shows the melodic lines for leaves one and five. The melody is composed with the purpose of giving each leaf a distinct character; that character is related to the scale with its tuning and with the time delays used with the phase canon.

Conclusion

The compositional approach I have described here places a great emphasis on strict procedures applied to both large-scale and small-scale parameters and to their coordination. Once these procedures were arrived at, their result was accepted without modification, and individual details were not altered to provide a "better fit." Large-scale design took precedence over local considerations in every aspect except for melody. This may be an extreme case, but it seems to me quite important that computer-generated music makes it possible to consider the way in which even the smallest acoustic detail relates to the whole. More than with other compositional media, with computers it is possible to integrate large- and small-scale design and to present a single, unifying scheme for an entire work. The geometric model I used for *Five-Leaf Rose* is such a scheme, but only one of many possibilities. Its primary attraction for me is that it is conceptually simple and yet can produce complex musical relationships.

References

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Grey, J. M. 1975. "An Exploration of Musical Timbre." Ph.D. dissertation, Department of Music, and technical report STAN-M-2. Stanford, California: Stanford University.

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Soundsheet Examples

Side One

Band One

Band One is an excerpt of *Mortuos Plango, Vivos Voco* by Jonathan Harvey.

Technical Notes

The excerpt starts at the beginning of the piece. The initial structure is the original recording of the tolling Winchester tenor bell (middle C) plus the 14 transpositions of the bell mapped onto the lower partials of the spectrum. They toll at the corresponding transposed tempi. To this are added recordings of the boy chanting the bell inscription on middle C and five partials, with proportionally faster tempi for each. At 50 sec, a re-enveloped synthetic bell is heard with a sequence of six others at spectrum-determined frequencies. These are phased with doubles of themselves, the phase being determined by another function. This yields an in-and-out-of-phase structure.

At 1 min 40 sec the second section starts; it is based on the partial G (781 Hz) which is announced by a transposed bell stroke (treated with certain speeds of beating). There are groups of consonants extracted from the text, with transposition patterns. An inverted bell envelope is heard which crescendos to the upper partials. At about 2 min the phoneme PRE ("Precis") is transposed from its vowel structure ε to a bell structure "sung" by synthetic boys whose partials then glissando to part of the bell spectrum of the next section (the fundamental is F at 347 Hz). Meanwhile, the vowel changes from ε to e—the vowel of the next section.

The third section, starting at 2 min 14 sec, is based almost entirely on glissando-modulations of the synthetic bell. Transposed bells punctuate the glissandi, and there is a loud peal of bells arpeggiating the lower partials of the tonic bell spectrum.

As the recording is a mixdown to stereo, the structure of the original eight-channel distribution has not been explained.

The work is published by Faber Music Limited, 3 Queen Square, London WC1N 3AU, United Kingdom.

Band Two

Featured are excerpts from *Dreamsong* by Michael McNabb, realized at the Center for Computer Research in Music and Acoustics (CCRMA), Stanford University, California. The entire work will soon be available on a record of music from CCRMA to be released by 1750 Arch Records, 1750 Arch Street, Berkeley, California 94709.

Technical Notes

Example 1. Time 30 sec to 100 sec: Noise source changing into soprano while panning along the path shown in Fig. 3. This is followed by the soprano chorus and amplitude modulation on the soprano chorus. The noise source plays the first three pitches given in Fig. 2b, and the chorus sings the first three notes of Fig. 2a.
Example 2. Time 220 sec to 250 sec: Bells transformed into soprano.
Example 3. Time 325 sec to 385 sec: Random formant instrument sings Fig. 2b, while the FM bells move in a circle. Figure 4 gives a graphic representation of this section.
Example 4. Time 515 sec to 550 sec: Crowd sound modulated by speaking voice.

Side Two

Band One

Excerpt from *Five-Leaf Rose* by Gary Kendall.

Technical Notes

In order to best illustrate the compositional techniques used in *Five-Leaf Rose*, the excerpt included here is of one complete section rather than of several smaller parts. Within this section one can hear many of the aspects described in my article such as the phase canon leaving and returning to synchronization, the slow modulation of timbres, etc. The one aspect of the composition which could not be retained in the stereo version is the spatial movement from front to back. This has been modified so that the voices crisscross in front.

This excerpt begins with the end of leaf 4 and continues through all of leaf 5. At the end of leaf 4 the phase canon has nearly returned to synchronization, the notes are in a middle register and the timbres are mellow and brasslike. Leaf 5 begins with a sudden discontinuity in both range and timbre. The range jumps up into a very high register and the timbres are bright and stringlike. As the fifth leaf unfolds the voices move farther out of synchronization so that by the middle of the leaf, a canon with a several-beat delay is clearly heard. The timbres vary considerably with the range of the notes; they modulate slowly between strings and muted brass. As the leaf reaches its conclusion, the voices return to synchronization and the timbres become similar to ones heard earlier in the piece.