

# A Particle-based Fuzzy Logic Approach to Sound Synthesis

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## Abstract

This article describes a novel audio synthesis technique based on sound particles, behavioral rules and fuzzy logic. Our approach borrows ideas from many-body classical physics, quantum mechanics, multi-agent and complex systems. The technique allows the creation of highly complex sonic behavior by very simple means, using the idea of a sound particle as its point of departure. Each sound particle in the system possesses several fuzzy properties, such as frequency, intensity and charge. These properties are used as inputs to a rule-based fuzzy logic inference system that controls the temporal evolution (trajectories) of the particles and the interactions between them. The rules are easy to elaborate, modify and adjust, because they resemble human reasoning and require no knowledge of the mathematical or physical nature of the trajectories or the interactions. The main objective of the proposed synthesis technique is to provide an intuitive, easy to use and flexible interface to the creation of very complex sonic phenomena.

## Introduction

Over the last decades, several sound synthesis techniques and algorithms have been proposed in the literature. Most of them are based on a wave-like view of sound and use standard mathematical or computational techniques such as Fourier series, digital filtering or physical modeling. The software interfaces to the synthesis algorithms are usually complicated, totally dependent on the underlying physical or mathematical model and do not allow for much flexibility and complexity. For musicians, it usually requires a significant level of technical knowledge to bring these techniques to optimum levels of use. Another aspect of these approaches is their relative simplicity in terms of how sound is constructed. Models are just simplifications of natural systems and synthesis algorithms that use mathematical or physical models constitute only approximations to the kind of real sounds that we find in nature. The natural world is in general highly complex and simplification inevitably signifies a loss of information, and in our case, musical interest.

We feel the necessity of having a more powerful and flexible approach to sound synthesis, one capable of handling complexity on a simple and powerful way, one that would allow musicians to create more complex sonic behavior with little effort. One of the ways to model complex systems is by the use of multi-agent environments or "particle systems", which are basically collections of a large number of agents (or particles) that interact within each other under certain condition or restrictions. Particle systems usually consist of large number of point masses (particles) moving under the influence of external forces such as gravity, vortex fields and collisions with stationary obstacles (Szeliski and Tonnesen, 1992). The basic entities in these kinds of systems differ from physical "particles" in that they already have an intermediate complexity themselves.

Although sound is considered to be wave-like in nature, certain scientists and composers have considered the idea that sound could also be represented and synthesized by dynamic particle systems. Several kinds of particle-based sound synthesis algorithms have been proposed in the literature (Roads, 2001; Sturm, 2001; Cook, 2002; Castagne and Cadoz, 2002). In general, Newtonian or stochastic equations govern the motion and collision of the point masses that produce the sounds and it is necessary to solve all the analytical differential equations and calculate the state for each particle before rendering a sample of sound. Such a process is computationally very expensive. Also, because analytical equations are a requisite, only certain types of dynamical systems can be considered. The behavior of complex and chaotic dynamical systems sometimes cannot be

described in terms of closed-form equations. Indeed, most non-linear systems are impossible to solve analytically (Strogatz, 1994).

## Fuzzy logic

Fuzzy logic is a concept derived from the mathematical branch of fuzzy sets (Zadeh 1965). In a narrow sense, it refers to a logical system than generalizes traditional two-valued logic for reasoning under uncertainty, allowing multiple values of truth. In a broad sense, it refers to all the theories and technologies that employ fuzzy sets (Yen and Langari 1999). In general, when fuzzy logic is applied to computers, it allows them to emulate the human reasoning process, quantify imprecise information and make decisions based on vague and incomplete data (Kosko 1993).

Fuzzy logic systems have been widely used in a variety of fields, most prominently engineering and control applications (Klir and Yuan 1995; Kosko 1993), but they have also been applied to other areas as diverse as data analysis (Bandemer and Gottwald 1995), economics, business and finance (Von Altrock 1997), sociology (Dimitrov and Hodge 2002) and geology (Demicco and Klir 2004). However, fuzzy logic has seldom been used in the artistic and creative fields, which is highly surprising, given that other related theories and techniques such as neural networks and genetic algorithms have been widely used for artistic purposes. In the specific case of music, despite the fact that Landy (2001) included fuzzy logic as one of the potential areas for tomorrow's music world, only a few applications related to creative activities have been reported in the literature.

## Crisp logic vs. fuzzy logic

In crisp logic (such as binary or boolean logic) variables are either true or false, black or white, 1 or 0. Aristotle's law of the excluded middle holds:  $A$  or not- $A$  (Kosko 1993). An object cannot be part of  $A$  and not- $A$  at the same time. An object cannot be a member of a set and its complement at the same time. If so, it's a contradiction. In contrast, fuzzy logic is defined with uncertain terms and partial values of truth are admitted. Things are not true or false or black and white anymore, they can be partially true or false or any shade of gray. In fuzzy logic, partial membership is something normal, an object can be a member of a set only partially, and as a consequence it can also be a member of the complementary set to a certain degree. Mathematically, fuzzy logic accepts values between 1 and 0.

## Fuzzy sets

Theoretically, a fuzzy set  $F$  of a universe of discourse  $X = \{x\}$  is defined as a mapping,  $\mu_F(x) : X \rightarrow [0, \alpha]$  by which each  $x$  is assigned a number in the range  $[0, \alpha]$ . When  $\alpha = 1$ , which is the usual, the set is called normal. In the extreme case where the distribution is of zero width, the membership function is reduced to singularities, i.e.; the fuzzy set reduces to a crisp set. If the singularities are of two possibilities, we have binary logic.  $\mu_F$  is the *grade of membership* (or degree of truth) of  $x$ . The most commonly used mathematical functions representing membership curves are triangular and Gaussian functions.

## Fuzzy systems

A *fuzzy system* is defined as a system with operating principles based on fuzzy information processing and decision making. In such a system both inputs and outputs are classified and de-classified into fuzzy sets. The process of classification is defined as *fuzzyfication* and the de-classification as *de-fuzzyfication*. Once the inputs are fuzzyfied several fuzzy rules are computed in parallel to produce the corresponding outputs. These outputs are then de-fuzzyfied so that the desired variables are obtained. A fuzzy system is capable of reproducing the same output for the same input parameters at any given time. Several methods and techniques for fuzzyfication and de-fuzzyfication are proposed in the literature, each one with its advantages and disadvantages.

## Fuzzy rules

Fuzzy logic attempts to emulate human reasoning with vague rules of thumb and common sense. Human beings make decisions based on rules. Even though we may not be aware of it, all the decisions we make are based on computer-like *if-then statements*. For example, if the weather is fine, then we may decide to go out. If the forecast says the weather will be bad today, but fine tomorrow, then we make a decision not to go today, and postpone it until tomorrow.

## Why go fuzzy?

Fuzzy systems are powerful and work in a way that resembles some characteristics of human behavior. Parallel computation of fuzzy rules reduces drastically the computation time compared to a traditional mathematical approach. Fuzzy systems allow approximation of highly non-linear systems with incredible accuracy. One of the nicest things about this is that it is not necessary to know any mathematical model in advance to approximate any system. Fuzzy logic allows us to build systems using common sense, and the fuzzy rules can be discussed, tuned, and detuned easily.

## A fuzzy logic approach to sound synthesis

The proposed approach considers sound particles that evolve dynamically in time, following complex trajectories in different planes or spaces. These trajectories (which define the interaction within particles and with external forces) are modeled by deterministic fuzzy inference rules. Parameters that describe the state of each particle on any given time are fuzzyfied and used as inputs to a fuzzy logic control system that affects the future state of the particles. The inference rules can be expressed in the form of simple linguistic expressions that related the inputs with the outputs of the fuzzy inference system.

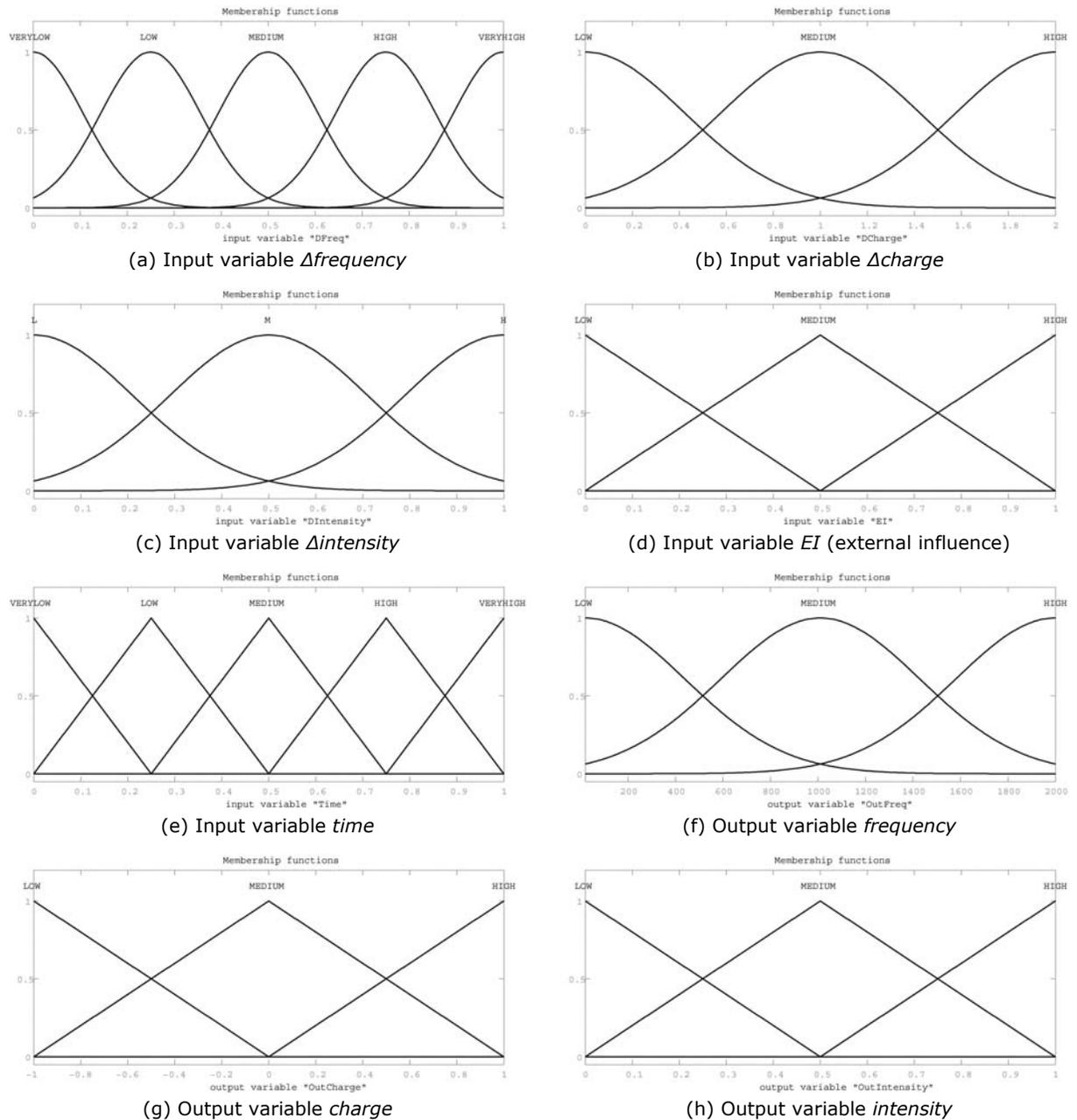
The main contribution of our approach is that by means of this fuzzy control system, we are able to specify highly complex, non-linear deterministic sound particle's trajectories in a very simple way, without the enormous amount of work that would be required to solve the analytical differential equations for a dynamical system. These highly complex non-linear trajectories, in turn, result in complex synthesized sounds.

## Parameters of the sound particles

In our model, each sound particle has the following properties:

- Fundamental frequency
- Intensity (amplitude)
- Charge
- External influence

In other words, each particle has a fundamental frequency and intensity at any given instant in time. Sound particles are also considered to be charged, like their atomic counterparts, but in a fuzzy way, ranging from a full negative charge of  $-1$  to a totally positive charge of  $+1$ . Also, each particle is influenced by external forces, such as the overall charge of the system, to a certain degree. All these properties are in fact fuzzy variables, which means that each one of them consists on several fuzzy sets or membership functions (usually three or five) and values of the variable can have partial membership to them. Figure 1 shows all the inputs and output of the proposed fuzzy inference system. Figure 1(a) shows the fuzzy variable  $\Delta frequency$ , which is defined as the normalized difference between the initial frequency of each particle and the actual frequency at any instant in time. In other words,  $\Delta frequency$  measures the deviation of each particle's main frequency from a given reference. In this case, five Gaussian membership functions were used, dividing the universe of discourse (ranging from 0 to 1) of  $\Delta frequency$  into five regions, labeled "VERY LOW", "LOW", "MEDIUM", "HIGH" and "VERY HIGH". Each value of the variable  $\Delta frequency$  belongs in a particular degree to each one of the five membership functions. For example, a  $\Delta frequency$  of 0.5 belongs a 100% to the "MEDIUM" region and a  $\Delta frequency$  of 0.6 belongs approximately 40% to the "MEDIUM" region and 60% to the "HIGH" region. Similarly, Figure 1(b) shows the membership functions of the variable  $\Delta charge$  and Figure 1(c) displays the fuzzy variable  $\Delta intensity$ , both defined in similar ways as  $\Delta frequency$ . The normalized variable EI (external influence), shown in Figure 1(d) contains three membership functions with a triangular shape. Figures 1(f), 1(g) and 1(h) display the output variables frequency (in Hertz), charge (between  $-1$  and  $+1$ ) and intensity (normalized). Figure 1(e) contains and additional input variable, time, which is also normalized. This adds the capability to have time-driven behaviors in the system, as opposed to behaviors only determined by particle's interactions.

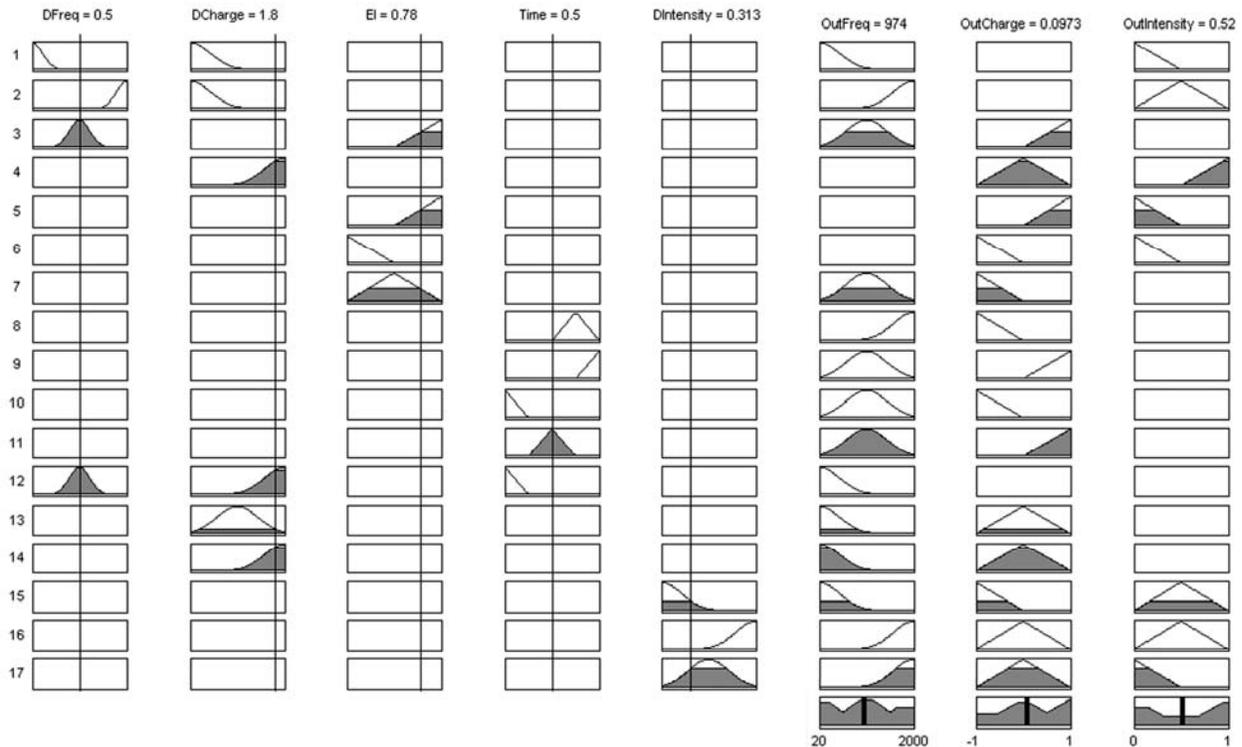


**Figure 1.** Fuzzy variables used as inputs and outputs to the fuzzy inference system.

As it was stated before, the trajectories of the particles are determined by several fuzzy inference rules, given in the form of simple linguistic statements. Examples of these rules could be:

- If  $\Delta frequency$  is VERY LOW and  $\Delta charge$  is LOW then  $frequency$  is LOW and  $intensity$  is LOW
- If  $\Delta charge$  is HIGH then  $charge$  is MEDIUM and  $intensity$  is HIGH
- If  $time$  is VERY HIGH then  $frequency$  is HIGH and  $charge$  is MEDIUM
- If  $\Delta frequency$  is MEDIUM and  $\Delta charge$  is LOW and  $EI$  is LOW then  $intensity$  is HIGH

All the fuzzy inference rules defined on a fuzzy inference system are computed in parallel in order to produce outputs in response to given inputs. Figure 2 shows a graphical representation of 17 rules used in the process of creating complex trajectories for the particles.

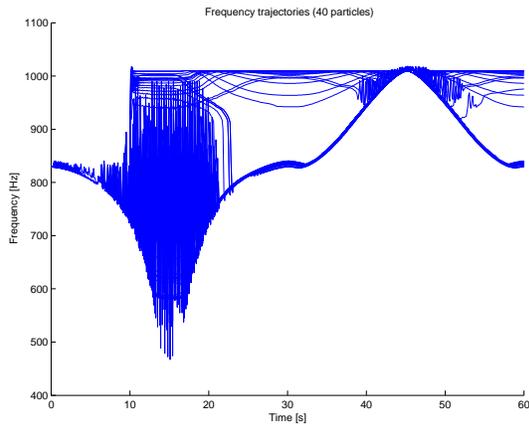


**Figure 2.** Graphical representation of the parallel computation of fuzzy inference rules.

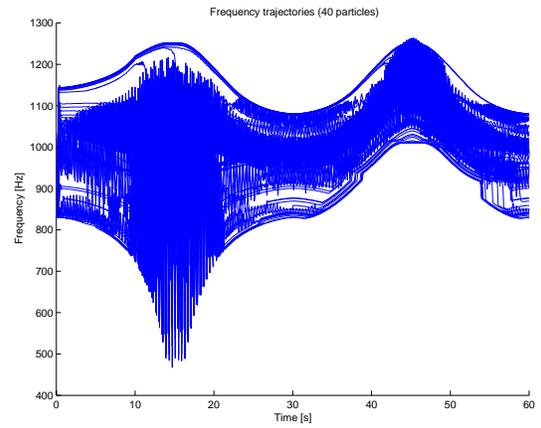
The five inputs to the system ( $\Delta frequency$ ,  $\Delta charge$ ,  $EI$ ,  $time$  and  $\Delta intensity$ ) are represented by the first five columns and the three outputs ( $frequency$ ,  $charge$  and  $intensity$ ) by the remaining three columns. In this particular case, Figure 2 shows the outputs produced by the input vector  $[0.5, 1.8, 0.78, 0.5, 0.313]$ . The parallel computation of the 17 rules produces the output vector  $[974, 0.0973, 0.52]$ .

Two sets of 40-particle systems were used to generate two 60-seconds mono audio files using the fuzzy inference system described by the variables shown in Figure 1 and the rules shown in Figure 2. In both cases, each particle was mapped to a sinusoidal wavetable digital oscillator. In the first set, all the particles had the same initial frequencies and intensities, but random initial charges. In the second set, all particles had the same initial frequencies and charges, but random initial intensities. Figure 3 shows the frequency, charge and intensity trajectories for the two sets. It is possible to appreciate that the trajectories that the particles follow are quite different for the two sets, although they have similar macro-shapes in relation to the flow time, due to the  $time$  variable that was added to the inputs to the system. In the case of Figure 3(a), corresponding to the first set, it is possible to observe several streams of trajectories that emerge as time progresses while in Figure 3(b) the particles follow a more chaotic behavior spread over a dense region of frequencies. Figure 3(c) shows trajectories where only negative charges occur, while in Figure 3(d) the trajectories also expand to the positive region of the charge domain. In Figure 3(e), a very chaotic behavior can be observed for the intensity trajectories in the middle of the time span, in contrast with Figure 3(f), where more dense and regular trajectories are followed by the particles. Overall, the different trajectories shown here are very complex and sometimes chaotic in nature. This complexity emerges as a result of the very simple fuzzy inference rules specified in Figure 2. This is an example of the power of fuzzy logic: with simple linguistic if-then like rules and the proper fuzzyfication of the parameters of interest, with no knowledge at all of the mathematical nature of dynamical systems, it is possible to generate very complex behavior.

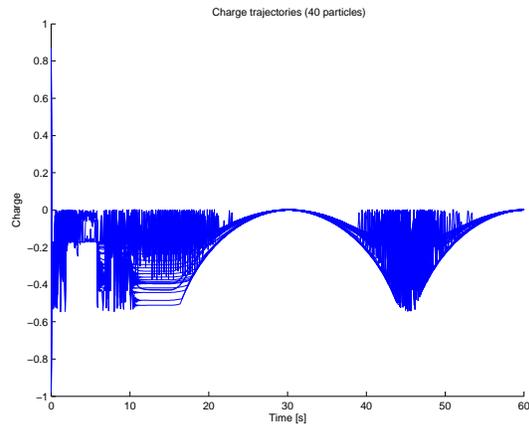
MP3 files containing the generated audio for the two different sets (set1 and set2) can be found inside the "CADIZ\_R\_sounds" folder included in the proceedings CD.



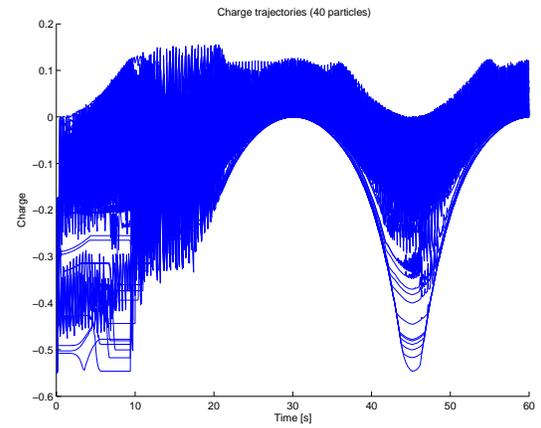
(a) Frequency trajectories (first set)



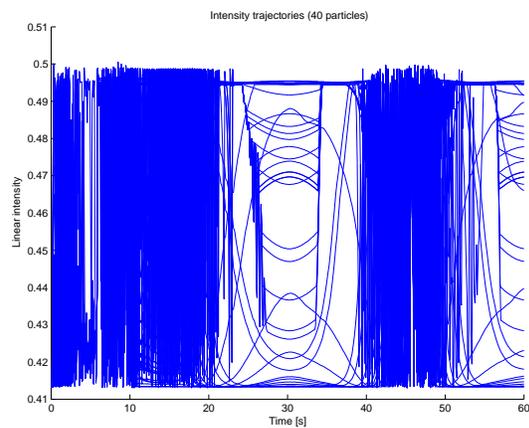
(b) Frequency trajectories (second set)



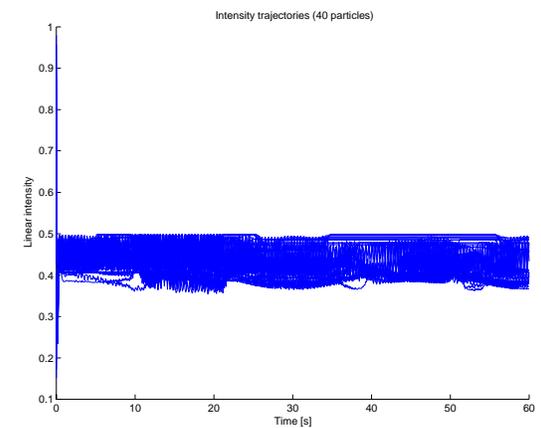
(c) Charge trajectories (first set)



(d) Charge trajectories (second set)



(e) Intensity trajectories (first set)



(f) Intensity trajectories (second set)

**Figure 3.** Frequency, charge and intensity trajectories for two sets of 40 particles with the same fuzzy inference system (inputs, outputs and rules) and different initial conditions.

## Conclusions and future work

This work had its origin in the creative necessity of composers for having new sounds at their disposal. The sounds that current synthesis techniques produce are widely known in the musical community. But in order to obtain different and fresh kinds of sounds, it is necessary to look for answers that go beyond the field and techniques of traditional music synthesis algorithms. In this particular case, the proposed approach requires also the application of ideas that come from disciplines apparently very far away from music.

We propose a novel and unexplored approach to sound synthesis and we hope to extend the available palette of computer generated sounds. The proposed approach is able to generate very complex non-random sonic phenomena with little effort. The described fuzzy logic system is simple to design and implement computationally.

One line of future work is to extend the number of parameters for the particles, especially parameters related to spatial positioning. This would allow the generation of stereo, multi-channel or spatial audio.

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## References

- Bandemer, H., and Gottwald, S. 1995. *Fuzzy Sets, Fuzzy Logic, Fuzzy Methods with Applications*. Chichester: John Wiley & Sons.
- Castagne, N. and Cadoz, C. 2002. Creating music by means of "physical thinking": the musicians oriented genesis environment. *5<sup>th</sup> International Conference on Digital Audio Effects (DAFX-02)*, pages 169-174, Hamburg, Germany.
- Cook, P. 2002. *Real Sound Synthesis for Interactive Applications*. Natick, MA: A.K. Peters.
- Demicco, R. and Klir, G. J. 2004. *Fuzzy Logic in Geology*. Elsevier Academic Press, Amsterdam, Boston.
- Dimitrov, V. and Hodge, B. 2002. *Social Fuzziology. Study of Fuzziness of Social Complexity*. Heidelberg, New York: Physica-Verlag.
- Klir, G. J., and Yuan, B. 1995. *Fuzzy sets and fuzzy logic: theory and applications*. Upper Saddle River, New Jersey: Prentice Hall.
- Kosko, B. 1993. *Fuzzy Thinking. The new science of fuzzy logic*. New York: Hyperion.
- Landy, L. 2001. "From algorithmic jukeboxes to zero-time synthesis: a potential A-Z of music in tomorrow's world (a conference provocation)". *Organised Sound*, 6(2):91-96.
- Roads, C. 2001. *Microsound*. Cambridge, MA: MIT Press.
- Strogatz, S. 1994. *Nonlinear Dynamics and Chaos*. Cambridge, MA: Westview.
- Sturm, B. 2001. Composing for an ensemble of atoms: the metamorphosis of scientific experiment into music. *Organised Sound*, 6(2):131-145
- Szeliski, R. and Tonnesen, D. 1992. Surface modeling with oriented particle systems. *Proceedings of the 19<sup>th</sup> Annual Conference on Computer Graphics and Interactive Techniques*, pages 185-194. ACM Press.
- Von Altrock, C. 1997. *Fuzzy Logic and Neurofuzzy Applications in Business and Finance*. Upper Saddle River, New York: Prentice-Hall.
- Yen, J. and Langari, R. 1999. *Fuzzy logic. Intelligence, Control and Information*. Upper Saddle River, New Jersey: Prentice-Hall.
- Ying, H., Ding, Y., Li, S., and Chao, S. 1999. "Fuzzy systems as universal approximators", *IEEE Trans. Syst., Man Cybern - Part. A: Syst. Hum.*, 29(5).
- Zadeh, L. A. 1965. "Fuzzy sets". *Information and Control*, 8:338-353.